Topological Invariants in Fixed Point Theory and Dynamical Systems
Gdańsk University of Technology, January 29–31, 2024

Expanding Lorenz maps with slope greater or equal to $\sqrt{2}$ are leo

Piotr Bartłomiejczyk[1] and Piotr Nowak-Przygodzki[2]

[1] *Faculty of Applied Physics and Mathematics, Gdańsk University of Technology*
[2] *Sopot*

We prove that expanding Lorenz maps with slope greater or equal to $\sqrt{2}$ are locally eventually onto (leo). To be more precise, recall that an *expanding Lorenz map* is a map $f\colon [0,1] \to [0,1]$ satisfying the following three conditions:

1. there is a critical point $c \in (0,1)$ such that f is continuous and strictly increasing on $[0,c)$ and $[c,1]$,
2. $\lim_{x\to c^-} f(x) = 1$ and $\lim_{x\to c^+} f(x) = f(c) = 0$,
3. f is differentiable for all points not belonging to a finite set $F \subset [0,1]$ and there is $\lambda > 1$ such that $\inf \{f'(x) \mid x \in [0,1] \setminus F\} \geq \lambda$.

Recall also that f is called *locally eventually onto* (*leo* for short) if for every opene set $U \subset [0,1]$ there is $n \in \mathbb{N}$ such that $[0,1] \setminus f^n(U)$ is finite. Assume that f is an expanding Lorenz map and $\beta = \inf \{f'(x) \mid x \in [0,1] \setminus F\}$. Let $f_0(x) = \sqrt{2}x + \frac{2-\sqrt{2}}{2} \pmod 1$. Our main result states that if $\beta \geq \sqrt{2}$ and $f \neq f_0$ then for every nonempty open subinterval $J \subset (0,1)$ there exists $n \in \mathbb{N}$ such that $f^n(J) \supset (0,1)$. In particular, f is leo.