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The Lyapunov exponent and rigorous computation of expansion in one-dimensional dynamics

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The Lyapunov exponent may be used as quantitative measure of sensitivity to initial conditions in dynamical systems. Positive Lyapunov exponent is often considered an indicator of chaotic dynamics.

We delve into the realm of one-dimensional dynamics and employ an approach that combines weighted directed graphs and interval arithmetic to achieve a method for rigorous computation of a lower bound for expansivity rate that is supposed to correspond to the Lyapunov exponent. We introduce a specific algorithm and accompanying software designed to serve this purpose.

In order to check the effectiveness of our methodology, we compare the results obtained with our algorithm against a non-rigorous numerical approximation of the Lyapunov exponent for the well known family of quadratic maps of the form $f_a(x) = a - x^2$. This does not only serve the purpose of checking the precision and validity of our computational framework, but also contributes to the understanding of limitations of a rigorous approach based on intervals in comparison to non-rigorous numerical simulations in the context of Lyapunov exponent estimation.