

# The Szymczak functor on the category of finite relations

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The Szymczak functor is a tool used to construct Conley index for discrete-time dynamical systems. A key property of the functor is that it establishes an equivalence relation between morphisms induced on index pairs and, hence, it enables the correct definition of the index. It follows from the features of the Szymczak functor that shift equivalence, whose equivalence classes are the isomorphism classes in the Szymczak category (i.e. target category of the functor), is the most general equivalence available. The universality of the Szymczak functor is also responsible for its computational weakness, because there is no general method to tell whether two objects in the Szymczak category are isomorphic or not.

In the case of dynamics modeled from data, the morphisms induced on index pairs are relations. In this talk, I will present an algorithmizable classification of shift equivalence classes for the category of finite sets with arbitrary relations as morphisms. Such a classification may provide a new method to study multivalued dynamical systems represented by relations.

The talk is based on:

M. Przybylski, M. Mrozek, J. Wiseman, The Szymczak Functor and Shift Equivalence on the Category of Finite Sets and Finite Relations, *J Dyn Diff Equat* (2023) <https://doi.org/10.1007/s10884-023-10332-9>