# On the twisted products of spheres that have the fixed point property

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A topological space $X$ is said to have the *fixed point property* if the equation
$$f(x) = x,\, x \in X$$
has a solution for every self-map $f$ of $X$. Familiar examples are:

a) If $X$ is the $n$-dimensional disk $D^n$ in the Euclidean space $\mathbb{R}^n$, then $X$ has the *fixed point property*.

b) If $X$ is the product $S^n \times S^n$ of two $n$-spheres, $n \geq 1$, then $X$ does not have the fixed point property.

A manifold is called a *twisted product* of two $n$-spheres $S^n$ if it is simply connected (i.e. $\pi_1 = 0$), and its cohomology agrees with $H^*(S^n \times S^n)$. Clearly, such manifolds are natural generalizations of the product $S^n \times S^n$, $n \geq 2$. Let $S(n)$ denote the set of all such manifolds . In this talk we

1) Give a homotopy classification of the set $S(n)$ of manifolds;

2) Determine the set $L(M) = \{L(f) \in \mathbb{Z}, f : M \to M\}$ of all possible Lefchetz numbers $L(f)$ for any $M \in S(n)$;

3) Find all manifolds $M \in S(n)$ for which $0 \notin L(M)$.

In contrast to b), the manifolds obtained in 3) are generalizations of $S^n \times S^n$ that enjoys the fixed point property.