Topological Invariants in Fixed Point Theory and Dynamical Systems
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On two-point BVPs in billiard spaces

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In the talk the two-point second order boundary value problem $\ddot{x}(t) = f(t, x(t))$, $x(0) = A$, $x(T) = B$ in an n-dimensional polyhedron K with absolutely elastic impacts on the boundary of K will be discussed. We call such set K, where solutions move and bounce off the boundary, a *billiard space*. Note that standard billiard problems can be modelled by a simple equation $\ddot{x}(t) = 0$. Hence the problems we are interested in can be described as a study of a movement of a ball on an uneven 'table' (n-dimensional). We shall focus on the existence and multiplicity of solutions, especially having first impacts only at $(n-1)$-dimensional faces of K. As a main tool the unfolding technique will be used for such spaces to reformulate the problem into a non-impulsive problem on $\mathbb{R}^n$ and to apply the Schauder fixed point theorem in this unfolded situation. As an illustration of the unfolding method and main results, the model of a system of two colliding balls on uneven ground is presented.

The talk is mainly based on the papers: G. Gabor and J. Tomeček, Multiple solutions of the Dirichlet problem in multidimensional billiard spaces, J. Fixed Point Theory Appl. 25 (2023), no. 1 and G. Gabor, Tessellation technique in solving the two-point boundary value problem in multidimensional billiard spaces, J. Math. Anal. Appl. 526 (2023), no. 1.