Topological Invariants in Fixed Point Theory and Dynamical Systems
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Predicting dynamical systems from embeddings with self-intersections

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How to model an unknown dynamics $T : X \to X$ by measuring a single observable $h : X \to \mathbb{R}$, i.e. having access to a time series

$$h(x), h(Tx), \ldots, h(T^m x)? \tag{1}$$

Classical Takens' time-delayed embedding theorems state that the original phase space X is typically injectively embedded into $\mathbb{R}^k$ via the map $x \mapsto \phi_h(x) = (h(x), h(Tx), \ldots, h(T^{k-1}x))$ provided that $k > 2 \dim X$. In this case one can uniquely reconstruct the dynamics on the time series (1) by embedding it into $\mathbb{R}^k$ via ϕ_h, and use it to approximate the dynamics. Such a model can be applied for predicting future values of (1).

Is it possible to faithfully model the dynamics or predict the future of (1) if one builds the model in a space of too small dimension and self-intersections occur in the embedding (e.g. when $k < 2 \dim X$)? Positive answers can be obtained if one adopts a probabilistic point of view, as conjectured by Schroer, Sauer, Ott and Yorke in 1998. I will present rigorous solutions to these conjectures and basics of a probabilistic Takens time-delayed embeddings theory. This is a joint project with Krzysztof Barański and Yonatan Gutman.