

# Detection of hidden periodicity in signals disturbed by additive non-Gaussian noise. Application to machine condition monitoring.

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We address the problem of detecting hidden patterns in signals that exhibit periodic correlation (PC); however, they are affected by non-Gaussian noise with unknown characteristics, a scenario commonly encountered in many areas of interest. Conventional approaches for identifying periodically correlated behavior typically operate in the time or frequency domain. Although our study incorporates these traditional methods, we enhance them by introducing robust alternatives to standard estimators of the autocovariance function and discrete Fourier transform.

Using robust tools, we develop improved versions of popular statistical methods that were originally intended to uncover hidden periodicity in “pure” PC models. Our work considers two types of PC model and two categories of additive noise, resulting in PC signals disturbed by non-Gaussian noise. Under such conditions, detecting hidden periodicity becomes significantly more challenging than in conventional scenarios. We apply our techniques to real-world datasets from the machine condition monitoring area, which is a main motivation for our research.

- [1] W. Żuławiński, J. Antoni, R. Zimroz, A. Wyłomańska: Robust coherent and incoherent statistics for detection of hidden periodicity in models with non-Gaussian additive noise, *EURASIP J. Adv. Signal Process.* **71**, 2024
- [2] W. Żuławiński, J. Antoni, R. Zimroz, A. Wyłomańska: Applications of robust statistics for cyclostationarity detection in non-Gaussian signals for local damage detection in bearings, *Mechanical Systems and Signal Processing* **214**, 111367, 2024

## The European Consortium for Mathematics in Industry: structure and activities

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The European Consortium for Mathematics in Industry (ECMI) is a consortium of academic institutions and industrial companies that acts cooperatively with the following aims:

- To promote and support the use of mathematical modelling, simulation, and optimization in any activity of social or economic importance.
- To educate Industrial Mathematicians to meet the growing demand for such experts.
- To operate on a European scale.

In this talk we will give an overview of the main activities carried out by ECMI and its members to achieve its objectives, ranging from the support and networking for research, to cooperation with industry, the editorial activities, the organization of workshops and conferences, etc. We will additionally focus on a set of recent success stories which arose from the cooperation between the ECMI members.

# GENEOnet: a transparent AI method for drug design based on Group Equivariant Non Expansive Operators

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joint work with Giovanni Bocchi<sup>1</sup>, Patrizio Frosini<sup>2</sup>, Alessandro  
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Equivariant operators are proving to be increasingly important in deep learning, in order to make neural networks more transparent and interpretable. The use of such operators corresponds to the rising interest in the so called “explainable artificial intelligence”, which looks for methods and techniques whose functioning can be understood by humans. In accordance with this line of research, Group Equivariant Non-Expansive Operators (GENEOs) have been recently proposed as elementary components for building new kinds of networks [1, 2]. Their use is grounded in Topological Data Analysis (TDA) and guarantees good mathematical properties to the involved spaces, such as compactness, convexity, and finite approximability, under suitable assumptions on the space of data and by choosing appropriate topologies.

In this talk we will show promising results obtained by applying GENEOs to protein pocket detection [3, 4]. Protein pockets detection is a key problem in the context of drug design, since the ability to identify a small number of potential binding sites, allows to speed up drug discovery procedures. In this talk we will show how GENEOs can be used to build a robust geometrical machine learning method, able to detect protein pockets better than ML techniques already in use, but being based only on 17 unknown parameters.

- [1] Bergomi, M. G., Frosini, P., Giorgi, D., and Quercioli, N.: Towards a topological-geometrical theory of group equivariant non-expansive operators for

- data analysis and machine learning. *Nature Machine Intelligence* (2019) doi: 10.1038/s42256-019-0087-3
- [2] Bocchi, G., Botteghi, S., Brasini, M., Frosini, P., and Quercioli, N.: On the finite representation of group equivariant operators via permutant measures. *Annals of Mathematics and Artificial Intelligence* (2023) doi: 10.1007/s10472-022-09830-1
  - [3] Bocchi, G., Frosini, P., Micheletti, A., Pedretti, A. *et al.*: GENEOnet: A new machine learning paradigm based on Group Equivariant Non-Expansive Operators. An application to protein pocket detection. (2022) preprint at arXiv:2202.00451.
  - [4] Bocchi, G., Frosini, P., Micheletti, A., Pedretti, A., Gratteri, C., Lunghini, F., Beccari, A.R., Talarico, C. GENEOnet: statistical analysis supporting explainability and trustworthiness. *Statistics*, 1–26, (2025). doi: 10.1080/02331888.2025.2478203

# Fractional modelling and physics-informed neural networks in epidemiology

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joint work with Jacky Cresson<sup>2</sup>

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Traditional compartmental models often lack the ability to reflect memory effects and anomalous transmission dynamics inherent in real-world epidemiological processes. In this presentation, we propose a framework that combines fractional-order differential equations with physics-informed neural networks (PINNs) to model the spread of infectious diseases. In the context of applications, we are particularly interested in domain-preserving neural networks.

Fractional derivatives introduce a non-local, history-dependent component that enhances the descriptive power of classical models, while PINNs provide a data-driven means of learning both the model dynamics and unknown parameters directly from epidemiological data. We demonstrate the effectiveness of our approach by applying it to dengue fever case data. This study highlights the potential of combining fractional calculus and neural differential solvers to improve both predictive accuracy and interpretability in epidemiological modeling.

- [1] J. Cresson, A.Szafrńska, *Discrete and continuous persistence problem - the positivity property and applications*, Communications in Nonlinear Science and Numerical Simulation Volume 44, March 2017, Pages 424-448.
- [2] Jacky Cresson, Anna Szafrńska. Neural networks preserving invariant domains and applications to population dynamics. In progress.
- [3] J. Cresson, A.Szafrńska, M. Péré, Numerical approach to the identification of fractional differential equations and applications in epidemics. Under review.

## When Heart Fibrosis Becomes a Disease

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Fibrosis is a hallmark of cardiac aging and is traditionally viewed as a physiological adaptation. In the sinoatrial node (SAN) and atrioventricular node (AVN), moderate fibrotic remodeling may stabilize tissue architecture. However, with progressive aging, the accumulation of fibrotic tissue and the loss of functional cardiomyocytes disrupt impulse generation and conduction.

Using a cellular automaton model, this study simulates age-related changes in the heart's conduction system. The results show that excessive fibrosis within the SAN impairs pacemaker activity, while fibrotic barriers in atrial tissue and around the AVN lead to conduction delays or blocks. These structural alterations contribute to the development of arrhythmias, particularly sinus bradycardia and atrioventricular block, both commonly observed in elderly individuals.

Where, then, lies the boundary between physiological remodeling and disease? Recognizing this transition is essential to understanding age-related cardiac dysfunction and may guide future therapeutic strategies focused on preserving conduction system integrity and cardiomyocyte viability.

- [1] Beata Jackowska-Zduniak. A Simplified Heart Age Model Based on Cellular Automata. Nunes-Gonzales J. David, Grana Manuel (red.): ***Modelling and Simulation'2024. The European Simulation and Modelling Conference*** (2024), 114–121.

## Educational Activities of ECMI

Cláudia Nunes<sup>1</sup>

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The aim of the ECMI Educational programme is to develop a student's mathematical and computational skills to solve industrial problems and development tasks in innovative ways. The ECMI Educational Programme was designed to educate industrial mathematicians to meet the growing demand for such experts. It comprises several initiatives, as the establishment of an ECMI Model Master. The ECMI Model Master in Mathematics for Industry has been running for many years in universities, and it is a model curriculum in Mathematics for Industry. Recently, ECMI has also established a model for a Master in Mathematical Data Science.

Other activities organized by ECMI include the ECMI Modelling Weeks and the ECMI Student Competition. ECMI has been running annual Modelling Weeks for students since 1988. Students come from all over Europe to spend a week working in small multinational groups on projects which are based on real life problems. The ECMI Student Competition is a more recent activity, and it is a competition involving teams of students of Bachelor, Master and PhD levels, where each team presents a solution to a mathematical problem of social or economic importance.

In this talk we will present these different educational activities that ECMI is organizing, as well as plans for the near future, where new ECMI members will be more than welcome.

Most of the information about these activities can also be found in the ECMI website:

<https://ecmiindmath.org/education/>

## Innovation and Product Positioning: When to Add or Replace

Cláudia Nunes<sup>1</sup>,  
joint work with Anne G. Balter<sup>2</sup>, Diogo Pereira<sup>1</sup>, Peter Kort<sup>2</sup>

<sup>1</sup>*CEMAT and IST*

<sup>2</sup>*Tilburg University*

We consider a monopolist firm, currently active in the market, which has the opportunity to innovate by introducing a new product. This involves decisions about the timing, quality, and positioning relative to the established product. To study this problem, we use the Hotelling line model. Moreover, we introduce demand uncertainty. The main result is that when the established product is sufficiently profitable, it will be replaced by the new product in the long run. This result is reinforced by demand uncertainty, which delays the new product introduction. Adding the new product alongside the established product so that they are both available for sale after the new product is introduced, may occur when the unit production cost of the new product is small. In the latter scenario, a hysteresis region may arise where the firm waits, while at lower demand levels, it immediately adds the new product alongside the established product, and at higher demand levels it immediately replaces the established product with the new one.



## Processing challenging data for machine learning

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Traditional techniques used for machine learning are not well fit to data, which are difficult, i.e. have a high dimensionality, imbalanced class distribution, are uncompleted, damaged, massive or are streaming in nature. Thus, searching for new and better techniques, methods or approaches suitable for working with such data is a really actual for the machine learning community. In my talk I will present selected research results on working with difficult data. Proprietary approaches for learning from big dataset, as well as, from imbalanced data and data streams will be presented. It will be also presented research results on methods for multi-label learning in the selected telecommunication system, where a problem of data damaged is observed.

## Fractional calculus in mathematical modeling

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joint work with Tomasz Stefański<sup>2</sup>

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The concepts of fractional order integral and derivative appeared in mathematics centuries ago. The history of applications of these concepts to other sciences is much shorter but is gaining more and more attention in recent decades. We are going to present some examples showing what stepping from integral-order to fractional-order models does to mathematical models.

## ML-driven analysis of anomalous diffusion

Janusz Szwabiński<sup>1</sup>

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In recent years, Single Particle Tracking (SPT) has become a popular method for quantifying the motion of individual particles in living matter with high spatial and temporal resolution. In a typical SPT measurement, the molecules of interest are tagged with fluorescent dye particles. When illuminated by a laser, the tags emit light and their positions can be determined using a microscope. By using lasers that flash at short intervals, the movement of the molecules can be tracked over time. The recorded positions are used to reconstruct the trajectories of individual molecules. These trajectories are then analysed to extract local physical properties of the molecules and their environment, such as velocity, diffusion coefficient (or tensor) and confinement (local density of obstacles).

The analysis of SPT trajectories is not a trivial task due to the stochastic nature of the molecules' motion. It usually starts with the detection of an appropriate motion type of a molecule, as this information may already provide insight into the mechanical properties of its environment. In this talk, several methods for identifying motion types will be presented, starting with the very popular technique based on the mean square displacement of particles, through statistical hypothesis testing and feature-based machine learning, and ending with deep learning classification methods.

## Topological data analysis enhances investigation of dynamical systems

Justyna Signerska-Rynkowska<sup>1</sup>

<sup>1</sup>*Gdańsk University of Technology*

Dynamical system theory can be successfully used in modelling and understanding of complex phenomena across various fields, including engineering, physics, and biology. A key concept in comparison and classification of dynamical systems is the notion of topological conjugacy. In the talk we will consider a problem of testing topological (semi-)conjugacy of two trajectories coming from unknown dynamical systems when only finite samples of those trajectories are given. In particular, we will discuss recently developed *ConjTests* methods, showing its scalability and robustness.

Moreover, as in data driven dynamics we often face significant challenges due to non-linearity and high-dimensionality, topological data analysis offers powerful technics to overcome these difficulties and get insight into the qualitative structure of dynamical systems. One noteworthy approach in this vein is the *Euler Characteristic Profile*. We will see that by capturing the underlying shape and connectivity of data, this method offers a robust framework for quantifying topological features and their evolution over time.

The talk is based on joint works ([1, 2]) with Paweł Dłotko, Michał Lipiński and Marta Marszewska.

- [1] P. Dłotko, M. Lipiński, J. Signerska-Rynkowska. Testing topological conjugacy of time series. *SIAM Journal on Applied Dynamical Systems* **23** (2024), 2939–2982. <https://doi.org/10.1137/23M1594728>
- [2] P. Dłotko, M. Marszewska, J. Signerska-Rynkowska. Topological characteristics of dynamics. (2025) (*in preparation*)

## Wrocław ECMI node: past, present and future

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Wrocław University of Science and Technology became a member of the European Consortium for Mathematics in Industry (ECMI) in 2007 and in 2014 was established as an ECMI Teaching Centre. We will outline a summary of past Wrocław activities, ongoing initiatives, and upcoming plans.

# Application of AI methods in predicting the likelihood of intracranial aneurysm rupture

Patryk Jasik<sup>1,8</sup>,

joint work with Julia Zakrzewska<sup>1</sup>, Justyna Fercho<sup>2,3,4</sup>, Piotr Fonferek<sup>1</sup>,  
Katarzyna Konieczna<sup>1</sup>, Hanna Lisowska<sup>1</sup>, Jakub Sadowy<sup>1</sup>, Daria Binerowska<sup>1</sup>,  
Maciej Pestka<sup>1</sup>, Michalina Dudra<sup>5</sup>, Weronika Jagieło<sup>5</sup>, Klaudia Kokot<sup>5</sup>, Dariusz  
Szplit<sup>6</sup>, Jacek Szypenbejl<sup>2</sup>, Tomasz Szmuda<sup>4,7</sup>, Mariusz Siemiński<sup>2</sup>

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Our study aims to go beyond the common conviction that assessing the likelihood of intracranial aneurysms rupture risk can be done only through the investigations of medical images [1]. We claim that preliminary estimations of the probability of intracranial aneurysm rupture risk are possible based on laboratory data and textual information from medical records [2]. In this study, we focus on feature extraction from medical records to improve models of rupture predictability. Machine learning and explainable AI methods are used to build classification models, offering insights into decision-making processes and enhancing the understanding of diagnoses. The ultimate goal is to develop a diagnostic tool to reduce mortality, hospitalisation costs, and neurological complications in patients with intracranial aneurysms. Assessing rupture risk is challenging, emphasising the need for early detection and prevention. Clinical data were collected from 2095 patients hospitalised with ruptured or unruptured aneurysms between 2006 and 2021 at the Neurosurgery Clinic of the University Clinical Center in Gdańsk. The dataset includes 997 unique laboratory tests and 83875 medical notes. Statistical and comparative analyses identified factors differentiating patients' groups with ruptured and unruptured aneurysms. Models built supported by features from laboratory tests and medical notes improved accuracy to 0.65, a 0.12 increase over models based only on laboratory tests. This shows how to improve the ML models by potentially using large language models for extracting valuable predictors from medical documents.

[1] Zhou Z et al. Classification, detection, and segmentation performance of image-based AI in intracranial aneurysm: a systematic review. *BMC Med Imaging*. **24** (2024) 164.

[2] Fercho J, et al. Application of artificial intelligence methods in predicting the likelihood of intracranial aneurysm rupture to support clinical decision-making. Submitted to *Computers in Biology and Medicine* (2025).

## Presentation of Dioscuri Centre in Topological Data Analysis

Paweł Dłotko<sup>1</sup>

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In this talk, I will present the research carried out at my Dioscuri Centre for Topological Data Analysis (<https://dioscuri-tda.org/>). Established under the auspices of the Max Planck Society, the Centre consists of approximately ten members, including the principal investigator, postdoctoral researchers, and PhD students. In addition, we have a number of permanent external collaborators in Poland, Europe, and the United States, and we regularly host seminars, schools for PhD students, conferences, scientific fairs, and public lectures.

We are engaged in a wide range of projects both in Poland and internationally, many of which are conducted in close collaboration with industrial partners. I will highlight our recent work in data analysis, statistical learning, materials science, and data visualization, illustrating it with several practical examples.

Finally, I will discuss opportunities for collaboration between the Centre and ECMI-associated organizations in Poland and across Europe.

## Faculty of Applied Physics and Mathematics of the Gdańsk University of Technology – from history through the present to the future.

Paweł Możejko<sup>1</sup>

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Gdańsk University of Technology with over 120 years of history is the oldest university in the Pomeranian Voivodeship and one of the oldest technical universities in Poland. Both Physics and Mathematics as research disciplines have been present at our University from the very beginning. The Faculty of Applied Physics and Mathematics of the Gdańsk University of Technology, despite its only 40-year history, is an active and interdisciplinary research center. Its scientific profile includes areas such as mathematics, physics, materials engineering and nanotechnology. International scientific cooperation is very important in our activities, especially in the context of projects financed by the European Union and long-term bilateral programs, among others, with institutions from Spain, Germany, Italy, Slovenia, Taiwan and China. These activities are strongly supported by the Gdańsk University of Technology, which is recognized as one of the leading research universities in Poland. By participating in the IDUB program (“Initiative of Excellence – Research University”) and the ENHANCE alliance supporting cooperation, interdisciplinary research and innovation, our university systematically strengthens its international position. The aim of the presentation is to present the Faculty of Applied Physics and Mathematics and its role in the broader mission of the Gdańsk University of Technology as a modern European research institution.



## Quantification of coupling between heart rate and blood pressure by means of entropy-based indices

Paweł Pilarczyk<sup>1</sup>,  
joint work with Grzegorz Graff<sup>2</sup>, José M. Amigó<sup>3</sup>,  
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In order to monitor mutual influence between heart rate (HR) and beat-to-beat blood pressure (BP) in patients, we propose an approach to the analysis of the pair of time series that is based on ordinal patterns and entropy.

Specifically, the two time series for each patient are aligned to each other in time, converted into ordinal patterns of a given length (e.g., 3 or 4), and then a collection of entropy-like indices is computed to measure the complexity of each time series, as well as their mutual relation. The indices are: permutation entropy, statistical complexity and self-transcript entropy computed for each individual time series; transcript entropy, mutual information, and transcript mutual information between the HR and BP series; all these indices also shifted in time by up to 9 positions.

We described this method in [1] and applied it to construct a classifier for distinguishing between healthy patients and those suffering from obstructive sleep apnea based solely on the measurement of their HR and BP taken during an outpatient exam conducted during the day.

- [1] P. Pilarczyk, G. Graff, J.M. Amigó, K. Tessmer, K. Narkiewicz, and B. Graff. Differentiating patients with obstructive sleep apnea from healthy controls based on heart rate–blood pressure coupling quantified by entropy-based indices. *Chaos* **33** (2023), 103140. <https://doi.org/10.1063/5.0158923>

# Why computers like Lorenz maps

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We consider how to check the properties of *topological transitivity* and *leo* (*locally eventually onto*) well-known in the theory of dynamical systems using a computer. We concentrate on the study of one dimensional dynamics in the class of Lorenz maps on an interval. We apply our numerical methods to the analysis of the 1D Courbage-Nekorkin-Vdovin model of a single neuron (see [1, 2]).

- [1] P. Bartłomiejczyk, F. Llovera, J. Signerska-Rynkowska. Spike patterns and chaos in a map-based neuron model. *Int. J. Appl. Math. Comput. Sci.* **33** (2023), 395–408.
- [2] P. Bartłomiejczyk, F. Llovera, J. Signerska-Rynkowska. Analysis of dynamics of a map-based neuron model via Lorenz map. *Chaos* **34** (2024), 043110.

# Fibrosis-Induced Arrhythmias: A Computational Approach to Micro-Reentries and Ectopic Foci

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Cardiac tissue is a prime example of a heterogeneous excitable system. Fibrosis is known to promote various types of cardiac arrhythmias by disrupting the normal propagation of electrical stimuli. In fibrotic tissue, conduction becomes more erratic and fragmented, leading to a noisy, breakable propagation pattern. The electrical wave follows a zigzag trajectory through a maze of non-conducting fibrotic regions.

We model cardiac tissue as a discrete system, incorporating random disconnections between cells to simulate the progressive growth of fibrosis. Monte Carlo simulations are then performed to determine the levels of fibrosis and ischemia at which the infarct region is most likely to behave as an ectopic focus. Our results reveal the emergence of micro-reentry and ectopic activity within micro-fibrotic regions, particularly near a critical topological threshold known as the percolation threshold. Using a simplified tissue model, we numerically compute electrocardiograms (ECGs) that successfully replicate experimental complex fractional atrial electrograms (CFAEs).

We apply such methodology to Patient-specific geometrical models of the ventricles, as well as infarct and peri-infarct zones generated from MRI images. Electrophysiological behavior is simulated using a human ventricular myocyte model [1]. We assess the minimal size of three-dimensional tissue slabs capable of sustaining micro-reentries under different stimulation protocols [2].

Our findings highlight the importance of the discrete topology of cardiac tissue. Notably, the proximity of the fibrosis fraction to the percolation threshold appears to be a key determinant in the formation of micro-reentries and the onset of certain arrhythmias.

- [1] R. Sachetto, S. Alonso, F. Otaviano Campos, B. M. Rocha, J. F. Fernandes, T. Kuehne, R. W. Dos Santos. Ectopic beats arise from micro-reentries near infarct regions in simulations of a patient-specific heart model. *Scientific reports* **8** (2018), 16392.
- [2] R. Sachetto, S. Alonso, R. W. Dos Santos. Killing many birds with two stones: hypoxia and fibrosis can generate ectopic beats in a human ventricular model. *Frontiers in Physiology* **9** (2018), 764.