Dimensional Causality: observation-only Bayesian inference of causal relations between dynamical systems

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The discovery of cause-and-effect relationships is a foundational element of scientific knowledge. Recently, we developed a new causality analysis method called Dimensional Causality (DC), which can detect and distinguish various fundamental forms of causal interactions between two dynamical systems. Specifically, DC can identify directed causal connections, circular or bidirectional interactions, and differentiate these from cases where two unrelated systems are influenced by a hidden common cause, or confounder [1]. To determine these relationships from observed time series, DC combines an embedding-based theoretical framework for dynamical systems with enhanced dimension estimation techniques [2] and Bayesian model inference. Our improved DC analysis utilizes optimized rules for parameter selection in dimension estimation. To assign likelihoods to each potential causal connection, we perform Bayesian integrals sensitive to both the perceived ordering and equality of dimension estimates. Although the efficacy of this method has been demonstrated on simulated dynamical systems, we here validate it with in vitro measurements, where the types of causal relationships are relatively well understood.

Local Field Potential (LFP) and Intrinsic Optical Signal (IOS) were recorded during in vitro experiments of evoked epileptic activity, induced by a magnesium-free Ringer solution [3]. In an earlier study, we demonstrated that Sugihara's cross-convergent mapping (CCM) method identified a directed causal influence from LFP to IOS activity, with no detectable signs of feedback. However, CCM could not rule out the possibility of a confounder and yielded ambiguous results without assuming a time delay for the effect. During the initial phase of evoked seizure-like activity, a clear unidirectional causal link from LFP to IOS ($\rightarrow$) is identified. However, as epileptic activity diminishes in later stages, the likelihood of a hidden common cause between LFP and IOS increases. Importantly, this causal direction was inferred without assuming any observable time delay.

Additionally, we performed an analysis for the COMPASS fusion experiment. Plasma density was measured using lithium beam emission spectroscopy (Li-BES) in parallel with magnetic field measurements from Mirnov coils. The DC method was applied to investigate the causality behind transient instabilities of plasma edge localized modes (ELMs) and their precursors. In plasma experiment number 17229, our method found a highly probable unidirectional relationship from the magnetic field to plasma density, though a common cause could not be excluded. Further measurements are planned to expand upon these findings.

These results demonstrate the DC method's applicability to real-world experimental data and provide valuable guidance for future applications.

Acknowledgements. Hungarian National Research, Development and Innovation Fund NKFIH K135837, NKFIH-FK 135285, the Hungarian National Brain Research Program KTIA NAP 2017-1.2.1-NKP-2017-00002 and the Hungarian Research Network (HUN-REN).

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