Nielsen coincidence number of (n,m)-valued pairs of maps of a circle

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Given sets X, Y and $n \in \mathbb{N}$, a map $f\colon X \multimap Y$ is *n-valued*, if for every $x \in X$ the image $f(x)$ has cardinality n [5]. If for every $x \in X$ the image $f(x)$ has cardinality at most n, then a map $f\colon X \multimap Y$ is *at most n-valued.* A *graph intersection point* of a pair of multivalued maps f and g is defined as a point $(x, y) \in X \times Y$ for which $f(x) \cap g(x) \neq \emptyset$ and the *Nielsen coincidence number* is defined as the number of essential graph coincidence classes. In this talk we consider (n, m)-valued pairs of maps $f, g\colon S^1 \multimap S^1$ and for them we determine the Nielsen coincidence number, using the degree of some at most nm-valued map.

[1] Brown R.F., Kolahi, K. *Nielsen coincidence, fixed point and root theories of n-valued maps.* J. Fixed Point Theory Appl. 14, 309–324 (2013).

[2] Brown R.F., Goncalves D.L. *On the topology of n-valued maps.* Adv. Fixed Point Theory, 8 (2018), 205-220.

[3] Brown R.F. *Fixed Points of n-Valued Multimaps of the Circle.* Bulletin of the Polish Academy of Sciences. Mathematics 54.2 (2006): 153-162.

[4] Maxwell, C. N. *The degree of multiple-valued maps of spheres*, 1978.

[5] Staecker P.C. *Axioms for the fixed point index of n-valued maps, and some applications.* J. Fixed Point Theory Appl. 20, 61 (2018).

Applying Sliding Window Techniques for Topological Data Analysis (TDA) of Temporal Geo-Data: Challenges and Optimizations

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[1] *Politecnico di Milano*

This talk introduces the initial scope of my thesis, which explores the use of sliding window techniques [1] for analyzing temporal geo-data. My work focuses on the Madagascar GDHY dataset, which includes crop yield data for four crops from 1981 to 2016, alongside the EVI dataset, which measures vegetation health. Each data point represents crop yield per hectare across Madagascar. I am currently employing geo-interpolation techniques to regularize the data resolution and using the Dask Python library to handle the computational challenges presented by the large dataset.

The main goal of the research is to apply Topological Data Analysis (TDA) to uncover complex patterns in the temporal data of crop yields. In this early stage, I am focusing on addressing computational challenges related to memory management and processing time, while optimizing the analysis process. This talk will discuss the methods being explored, the challenges faced, and the potential strategies for improving the efficiency and scalability of the analysis.

[1] Anish Rai, Buddha Nath Sharma, Salam Rabindrajit Luwang, Md.Nurujjaman, Sushovan Majhi. Identifying Extreme Events in the Stock Market: A Topological Data Analysis. *Chaos* **34** (2024), 10. `https://doi.org/10.48550/arXiv.2405.16052`

Analysis of global dynamics using Conley-Morse graphs

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I will talk about about a method for creating a database of global dynamics with the help of Conley-Morse graphs. It lets us create a map on a rectangular grid for selected ranges of parameters that is helpful for identifying interesting, non-trivial dynamics and qualitative changes of dynamics.

The method utilises combinatorial approach to dynamics, Morse sets and the Conley index. Therefore, it was possible to create a software that will carry out the computations for discrete- and continuous-time systems in a rigorous manner. The software is available here, together with a guide how to compile the software.

The method was used in [4] for an analysis of a two-dimensional model of a neuron introduced by Chialvo in 1995. We plan to use the method to further the research done in [5] about the discrete two–gene Andrecut–Kauffman model. I will also use the method in my Master's thesis to analyse tritrophic food chain models.

$$\text{Model A} \quad \begin{cases} x' = a(x - x_0) - \alpha_1 xy, \\ y' = -by + \alpha_1 xy - \alpha_2 yz, \\ z' = -c(z - z_0) + \alpha_2 yz, \end{cases} \tag{1}$$

where a, b, c, x_0 and z_0 are positive parameters, and

$$\text{Model B} \quad \begin{cases} x' = rx(1 - px) - \frac{\alpha_1 x}{1+k_1 y} y, \\ y' = -by + \frac{\alpha_1 x}{1+k_1 y} y - \alpha_2 yz, \\ z' = -c(z - z_0) + \alpha_2 yz, \end{cases} \tag{2}$$

where r, p, b, c, x_0 and z_0 are positive parameters.

[1] Conley-Morse Graphs Computation software https://www.pawelpilarczyk.com/cmgraphs/

[2] Kokubu H., Spendlove K. A User's Guide to the Conley-Morse Database. https://kellyspendlove.github.io/cmdbSurveyII.pdf

[3] Arai, Z. et al., A Database Schema for the Analysis of Global Dynamics of Multiparameter Systems, *SIAM J. Appl. Dyn. Syst.* **Vol. 8**, No. 3 (2009), 757–789. https://doi.org/10.1137/080734935

[4] Pilarczyk P., Graff G., Signerska-Rynkowska J. Topological-numerical analysis of a two-dimensional discrete neuron model. *CHAOS* **33** (2023), 043110. https://doi.org/10.1063/5.0129859

[5] Rosman M., Palczewski M., Pilarczyk P., Bartłomiejczyk A. Bistability and chaos in the discrete two-gene Andrecut-Kauffman model. https://doi.org/10.48550/arXiv.2411.16699

Sequences realizable by diffeomorphisms of closed surfaces

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A sequence of non-negative integers $(\varphi_n)_{n=1}^{\infty}$ is called realizable if there exists a map f of a set X, $f\colon X \to X$ such that φ_n is equal to the number of fixed points of the n-th iteration of f.

In [1], Windsor conducted the construction in which arbitrary realizable sequence $(\varphi_n)_{n=1}^{\infty}$ was realized by a diffeomorphism on torus. It raises the question of which sequences are realizable for different mappings on closed surfaces.

In this presentation, I will briefly describe the mentioned construction of a diffeomorphism on torus and I will discuss whether it is possible to construct similar maps on other compact 2-manifolds without boundary.

[1] A.J. Windsor, *Smoothness is not an obstruction to realizability*, Ergod. Th. & Dynam. Sys. (2008), 1037–1041 `https://doi.org/10.1063/5.0158923`

Predicting mechanical properties of porous materials using topological data analysis

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joint work with Rafał Topolnicki[1], Michał Bogdan[1], Paweł Dłotko[1],
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Porous metals are increasingly important in technology. Due to their tunable mechanical properties, they are promising candidates in various emerging applications such as metallic scaffolds for load-bearing bones and lightweight structures for transport technologies. The aim of this study is to create topological descriptors of porous materials that allow a fast prediction of their mechanical properties. At present, the main focus is on Young's modulus. The topological properties of an object do not change when the object is rotated, while Young's module may depend on direction. To construct direction-aware descriptors, we encoded direction-dependent information in filtration values. We combine topological data analysis with theoretical models based on material porosity for better results. In this talk, we will present new topological descriptors of porous materials and discuss the effectiveness of regression models based on them.

Generalized configuration space and its homotopy groups

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joint work with Xuezhi Zhao[2]

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[2] *Capital Normal University*

Let M be a subset of vector space or projective space. The authors define the *generalized configuration space* of M which is formed by n-tuples of elements of M where any k elements of each n-tuple are linearly independent. The *generalized configuration space* gives a generalization of the classical configuration space defined by E.Fadell. Denote the *generalized configuration space* of M by $W_{k,n}(M)$. The authors are mainly interested in the calculation about the homotopy groups of generalized configuration space. This article gives the fundamental groups of generalized configuration spaces of $\mathbb{R}P^m$ for some special cases, and the connections between the homotopy groups of generalized configuration spaces of S^m and the homotopy groups of Stiefel manifolds. It is also proved that the higher homotopy groups of generalized configuration spaces $W_{k,n}(S^m)$ and $W_{k,n}(\mathbb{R}P^m)$ are isomorphic. This is a joint work with Xuezhi Zhao.

[1] Jun Wang and Xuezhi Zhao. On generalized configuration space and its homotopy groups. *Journal of Knot Theory and Its Ramifications* **29** (2021), 2043001. `https://doi.org/10.1142/S0218216520430014`

Dold sequences for group actions

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joint work with Jacek Gulgowski[2]

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Classical Dold sequences $a_n = |Fix(f^n)|$, which enumerate fixed points of iterated maps, are fundamental in understanding periodic points and dynamical systems. My presentation extends this concept to group actions.

A group G acting on a space X is defined by a map $G \times X \to X$ satisfying appropriate conditions. Key elements in this framework include stabilizers $Stab(x)$ of the point $x \in X$, the set $Fix(H)$, which is the set of points stabilized by a subgroup $H \subseteq G$, and $Orb(H)$, which represents the orbit set for H. An important result in this field is the Orbit- Stabilizer Theorem. It establishes a relationship between the orbit cardinality and subgroup properties under specific condition. By focusing on finite-rank subgroups of G, we can define a Dold sequence $a_H = |Fix(H)|$ indexed by the finite rank subgroups of G. This framework not only unifies and extends the classical results but also provides new insights into the structure of the set of periodic points.

Surface homeomorphisms of algebraically finite type

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Adam Mickiewicz University, Poznań

In [3] Nielsen investigated properties of surface mapping classes of algebraically finite type, defined to be represented by homeomorphisms that are either periodic or reducible and periodic outside an invariant system of circles. In other words, they have no pseudo-Anosov pieces in Nielsen–Thurston decomposition. The name "algebraically finite type" was motivated by Nielsen's conjecture that such classes can be defined purely algebraically as the ones that induce a map on the first homology group whose spectrum consists only of roots of unity (the latter classes are called quasi-unipotent). These two definitions do not coincide because of Thurston's construction of pseudo-Anosov map inducing the identity transformation. However, it is still an open question which symplectic transformations can be obtained from mapping classes of algebraically finite type. We will discuss this problem, important also from the point of view of dynamics. Da Rocha [1] showed that the classes containing Morse–Smale diffeomorphism and classes of algebraically finite type are the same. Some constructions in terms of Lefschetz numbers we provided in [2].

[1] L. F. da Rocha, *Characterization of Morse—Smale isotopy classes on surfaces*, Ergod. Th. & Dynam. Sys. 5 (1985), 107–122.

[2] G. Graff, W. Marzantowicz, Ł. P. Michalak, A. Myszkowski, *Every finite set of natural numbers is realizable as algebraic periods of a Morse–Smale diffeomorphism*, preprint (2024), arXiv:2408.12372.

[3] J. Nielsen, *Surface transformation classes of algebraically finite type*, Danske Vid. Selsk. Mat.-Fys. Medd. 21 no. 2 (1944), 89 pp.

Periodic solutions of Hamiltonian system on CP^N

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January 5, 2025

Arnold conjecture for CP^N states that Hamiltonian system generated by $H \in C^\infty(S^1, CP^N)$ has at least $N+1$ solutions with period 1. This result has been proven many times. In [1], authors did it using Conley's index. Based on their approach it is possible to prove this conjecture with additional condition using Leray-Schauder's degree.

In this presentartion, I will give an overview of this proof and later I will talk about extention of this result to hamiltonian delay equations and Arnold conjecture for $CP^N \times CP^M$.

[1] Asselle, L., Izydorek, M., Starostka, M. (2023). The Arnold conjecture in CP^n and the Conley index. *DISCRETE AND CONTINUOUS DYNAMICAL SYSTEMS-SERIES B* 28, 2603–2620. `https://doi.org/10.3934/dcdsb.2022184`

A Quantitative Method for Analyzing Dynamical Systems Using Topological Data Analysis Tools

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joint work with Paweł Dłotko[2], and
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In this presentation, I will discuss preliminary results from the analysis of both sampled and continuous dynamical systems using topological data analysis techniques. The core method employed is the Euler Characteristic Profile (ECP) [1], which is applied to vector fields sampled from the phase space. This robust characteristic of vector fields allows us to establish a distance metric between different types of dynamics, whether continuous or discrete, even when only finite samples are available. The proposed suite of methods serves as an alternative to traditional Conley index theory, with the added benefit of stability under small perturbations of the vector field.

Our initial research is focused on examining two- and three-dimensional dynamics, including features like fixed points, saddle points, periodic orbits, multistability, and chaotic behaviors. The goal is to explore their organization within the space defined by the corresponding Euler profiles. This work holds the potential to offer a novel perspective on the qualitative analysis of dynamical systems.

[1] P. Dłotko, D. Gurnari. Euler characteristic curves and profiles: a stable shape invariant for big data problems. *GigaScience* (2023). `https://doi.org/10.1093/gigascience/giad094`

Stability of Cellular Automata

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joint work with Jacek Gulgowski[2]

[1,2] *University of Gdańsk*

Our research into stability of cellular automata involves a so called Lyapunov exponent, introduced in 1992[1]. The approach allows one to measure the influence that a defect has on the evolution of an automaton. We aim to obtain values for the elementary 256 binary CA of local range 1. The complexity of the task varies. Generally, the defect path can take different forms depending on the initial configuration, thus the exponent of a given CA may have several values with certain probabilities. We would like to develop a level of understanding of the topic in order to introduce a valid measure and tackle more complicated or higher dimensional examples of cellular automata.

[1] F. Bagnoli, R. Rechtman, S. Ruffo. Damage spreading and Lyapunov exponents in cellular automata. *Physics Letters A* **172, Issues 1–2** (1992), 34–39. `https://doi.org/10.1016/0375-9601(92)90185-O`

Topological Tools for Phase Transitions: Exploring the 2D Ising Model

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Phase transitions are important phenomena where system properties change significantly as external parameters, like temperature, vary. While traditional methods often rely on Hamiltonians to identify critical points, not all systems allow for a clear Hamiltonian definition. This talk presents a topological approach to studying phase transitions, using the 2D Ising model as a case study.

By running Monte Carlo simulations with Metropolis and Wolff updates, I analyze spin configurations at different temperatures. This helps reveal how topological features change near the critical temperature. The goal is to develop a universal framework for detecting phase transitions, even in systems without explicit Hamiltonians.

This work sets the stage for future studies, including applications to higher dimensions or more complex models.

Topology meets mechanics: predicting the response to compression in metallic porous materials using Fourier-based computational approaches and topological data analysis

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joint work with[1], Rafał Topolnicki[1], Sergei Zorkaltsev[2], Jakub Malinowski[1], Bartosz Naskręcki[3], Maciej Harańczak[2], Paweł Dłotko[1]

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Metallic porous materials are used in wide-ranging practical applications, from lightweight metal structures to medical implants. However, their use in a given application depends on obtaining the desired mechanical properties, such as Young's modulus and yield stress. Advances in 3D printing enable creating a wider choice of prescribed structures than ever before. However, to take ad- vantage of this opportunity one must predict which structures will exhibit the desired mechanical properties. Instead of testing each candidate structure di- rectly using the computationally expensive simulation methods, our strategy rests on using Fast Fourier Transform-based computational methods to create a database of sample structures that will be used to build predictive models based on variables defined by topological data analysis (TDA) whose results can be extrapolated to a wider space of possible structures.

Persistent Homology of Maps and Relations

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The study of persistent homology has provided profound insight into the topology of data, but its extension to broader mathematical constructs remains an ongoing challenge. In this talk, we discuss the persistent homology of maps and relations, presenting a generalized framework that extends the study of persistent structures from self-maps to broader classes of relations.

Building on the foundational work of Edelsbrunner, Jabłoński, and Mrozek (2015) [1], which laid the foundation for the analysis of persistence in self-maps, and incorporating the framework introduced by Harker, Kokubu, Mischaikow and Pilarczyk (2016) [2] for relations, we present an approach to understanding the persistence of relations.

This extension of persistent homology opens the door to a wide range of applications in fields such as computational topology, dynamical systems, and topological data analysis. Finally, we outline future directions for this research, including the development of computational methodologies.

[1] H. Edelsbrunner, G. Jabłoński, M. Mrozek. The persistent homology of a self-map. *Foundations of Computational Mathematics* **15** (2015), 1213—1244. `https://doi.org/10.1007/s10208-014-9223-y`

[2] S. Harker, H. Kokubu, K. Mischaikow, P. Pilarczyk. Inducing a map on homology from a correspondence. *Proceedings of the American Mathematical Society* **144** (2015), 1787—1801. `https://doi.org/10.1090/proc/12812`

Bistability and Chaos in the Discrete Two–Gene Andrecut–Kauffman Model

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joint work with Michał Palczewski[1,2], Paweł Pilarczyk[1,3],
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[4] *BioTechMed Center, Gdańsk University of Technology, ul. Narutowicza 11/12, 80-233 Gdańsk, Poland.*

We conduct numerical analysis of the 2–dimensional discrete–time gene expression model originally introduced by Andrecut and Kauffman [1]. In contrast to the previous studies, we analyze the dynamics with different reaction rates α_1 and α_2 for each of the two genes under consideration. We explore bifurcation diagrams for the system with α_1 varying in a wide range and α_2 fixed. We detect chaotic dynamics by means of the positive maximum Lyapunov exponent and we scan through selected parameters to detect those combinations for which chaotic dynamics can be found in the system. Moreover, we find bistability in the model, that is, the existence of two disjoint attractors. Both situations are interesting from the point of view of applications, as they imply unpredictability of the system. Finally, we show some specific values of parameters of the model in which the two attractors are of different kind (a periodic orbit and a chaotic attractor) or of the same kind (two periodic orbits or two chaotic attractors).

[1] M. Andrecut and S. Kauffman. Chaos in a discrete model of a two-gene system. *Phys. Lett. A* **367** (2007), 281—287. `https://doi.org/10.1016/j.physleta.2007.03.074`

Chaotic itinerancy in globally coupled logistic maps

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joint work with Paweł Pilarczyk[1]

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Chaotic itinerancy (CI) is a type of dynamical behavior observed in high-dimensional dynamical systems. In CI, orbits are attracted to an ordered motion state and stay there for a while. Eventually, they depart from the ordered state and enter into high-dimensional chaotic motion. After some time, they once again reach an ordered state and this process continues. These ordered motion states are called attractor ruins. We analyze the transitions between attractor ruins that occur without any clear pattern.

This phenomenon has been observed across various domains, including neural dynamics and optical turbulence [1]. In this talk, I will describe a globally coupled logistic map, which is a simple prototype model for chaotic itinerancy in a symmetric dynamical system.

[1] Kaneko, K., & Tsuda, I. (2003). *Chaotic itinerancy.* Chaos, 13(3), 926-936

Fixed point indices of iterates of an orientation-reversing homeomorphism at a fixed point which is an isolated invariant set

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joint work with Grzegorz Graff[2]

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Let f be an orientation-reversing homeomorphism of $\mathbb{R}^m$ ($m \geq 3$) with an isolated fixed point at 0 for each iterate of f. In [2], the authors examined all possible sequences of indices $\{\mathrm{ind}(f^n, 0)\}_{n=1}^{\infty}$ for $m = 3$, under the additional assumption that $\{0\}$ is an isolated invariant set. They proved that, in this case, the indices have highly constrained forms.

In a more recent work [1], we show that if the assumption requiring $\{0\}$ to be an isolated invariant set is removed, the indices are bounded only by the Dold relations. Moreover, using our construction, we can realize all the forms determined in [2] such that $\{0\}$ is an isolated invariant set.

Building upon the techniques outlined in [1], we delve into an open problem concerning the possible forms of the sequence $\{\mathrm{ind}(f^n, 0)\}_{n=1}^{\infty}$ under the assumption that 0 is an isolated invariant set and $m > 3$. This discussion is intended to expand on the result established in [2].

[1] Grzegorz Graff and Patryk Topór. Fixed point indices of orientation-reversing homeomorphisms. *arXiv:2409.08753* (2024), `https://doi.org/10.48550/arXiv.2409.08753`

[2] L. Hernńdez-Corbato, P. Le Calvez and F. R. Ruiz del Portal. About the homological discrete Conley index of isolated invariant acyclic continua. *Geometry & Topology* **17** (2013), 2977–3026, `https://doi.org/10.2140/gt.2013.17.2977`

TDA meets dynamics

Paweł Dłotko[1]

[1] *Dioscuri Centre in Topological Data Analysis*

In this presentation, I will provide an overview of topological data analysis (TDA) methods that are particularly well-suited for analyzing dynamical data, with a focus on sampled dynamics. The discussion will start with an introduction to persistent homology, a key technique for differentiating the state spaces of various dynamical systems. We will then delve into more advanced methodologies, including Euler characteristic curves and profiles integrated with goodness-of-fit tests. To conclude, I will present conjugacyTests, a set of methods designed to evaluate the conjugacy of two finitely sampled trajectories. This talk will highlight foundational tools and illustrate their applications within dynamical systems theory.

Parametrized topological complexities of maps

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[2] *Institute of Mathematics, Physics and Mechanics, Ljubljana*

A manipulation plan for a robotic device can be viewed as a section of certain projection associated to the forward kinematic map from the configuration space of a robot to its work space. As it is in general impossible to find a manipulation plan depending continuously on the input data, one defines the topological complexity of a manipulation problem as the minimal number of continuous partial sections needed to cover all possible instances of input data. There are several variants of this concept, developed by Pavešić, Murillo and Wu, Scott and other authors, cf. [1]. In this talk we will discuss a parametrized version of topological complexity of a map which allows a treatment of the manipulation planning problem under varying conditions.

[1] P. Pavešić. *Topological complexity of a map.* In: Topology and AI, Michael Farber and Jesús Gonzáled (eds.), EMS Series in Industrial and Applied Mathematics 4 (EMS Press, Berlin, 2024), pp. 335-362.

Itineraries of nonoverlapping generalized Lorenz-like maps

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joint work with Piotr Nowak-Przygodzki[2]

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Our result states that if the rotation number of a nonoverlapping generalized Lorenz-like map is rational then all points in $[0, 1)$ have eventually periodic itineraries with the same specific ordering of the periodic part of the itinerary uniquely determined by the rotation number. To be more precise, recall that a *generalized Lorenz-like map* is a map f of an interval $I = [0, 1)$ to itself, for which there exists a point $c \in (0, 1)$ such that f is nondecreasing on $I_L = [0, c)$ and on $I_R = [c, 1)$. If $f(0) \geq f(1^-)$ then the generalized Lorenz-like map f is called *nonoverlapping*. The *itinerary* of x under f is the sequence $(s_0 s_1 s_2 \dots)$ where

$$s_j = s_j(x) = \begin{cases} 0 & \text{if } f^j(x) \in I_L, \\ 1 & \text{if } f^j(x) \in I_R. \end{cases}$$

For a point $x \in [0, 1)$ and a positive integer n we will denote by $R(x, n)$ the number of integers $i \in \{0, \dots, n-1\}$ such that $f^i(x) \in I_R$. If the limit $\rho(x) = \lim_{n\to\infty} \frac{R(x,n)}{n}$ exists, we will call it the *rotation number* of x. Note that if f is a nonoverlapping (i.e., $f(1^-) \leq f(0)$) generalized Lorenz-like map, then all points have the same rotation number and we will denote it by $\rho(f)$ (see [2]). We are ready to formulate our theorem.

Let f be a nonoverlapping generalized Lorenz-like. Assume that $\rho(f) = p/q$, where p and q are coprime nonnegative integers. Then for all $x \in [0, 1)$ there is $m \in \mathbb{N}$ such that for all $k \geq 0$ we have

$$s_{m+k}(x) = 0 \text{ if and only if } kp \pmod{q} < q - p.$$

[1] Piotr Bartłomiejczyk, Piotr Nowak-Przygodzki. Itineraries of nonoverlapping generalized Lorenz-like maps. *preprint.*

[2] F. Rhodes, Ch. L. Thompson. Rotation numbers for monotone functions on the circle. *J. London Math. Soc.* **34** (1986), 360–368.

Dynamics of homeomorphisms of surfaces

Wacław Marzantowicz joint work with ...

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The dynamics of a self-map $f : X \to X$ is described by invariants as the sequence of numbers of periodic points $\{|P^n(f)|\}$, $P^n(f) = \mathrm{Fix}(f^n)$, the set of minimal periods $Per(f) \in \mathbb{N}$, or the topological entropy $\mathbf{h}(f) \in \mathbb{R}^+ \cup \{0\}$. In the case when $X = S_{\mathrm{g}}$ is a an oriented surface of genus g, $\mathrm{g} \geq 2$, and f is a homeomorphism we can use many topological tools due to the Nielsen-Thurston classification theorem. It states that up to a homotopy, thus isotopy, every homeomorphism f of S_{g} is represented by: either a periodic homeomorphism, or a pseudo-Anosov homeomorphism, or a reducible homeomorphism. In the last case each piece of the reduction can be periodic, or pseudo-Anosov. It is important, but difficult, to distinguish when f is the pseudo-Anosov or its reduction contains at least one pseudo-Anosov piece, because then $Per(f)$ is infinite, e.g. there is infinitely many periodic points, and $\mathbf{h}(f) = \log(\lambda_f)$ where λ_f is the largest stretching (expanding) factor of the pseudo-Anosov piece. From the Nielsen theory we know that $\lambda_f = N^\infty(f) := \limsup \sqrt[n]{N(f^n)}$, but an affective computation of the latter is also strenuous. In last two decades, to resolve this difficulty there was developed a theory which let us to express λ_f as the as the spectral radius of $H_1(\tilde{f})$, where $H_1(\tilde{f}) \in Sp(2\mathrm{g}, \mathbb{Z})$ is the linear map induced on $H_1(\tilde{S}_{\tilde{\mathrm{g}}}; \mathbb{Z}) = \mathbb{Z}^{2\tilde{\mathrm{g}}}$ by a homeomorphism $\tilde{f} : \tilde{S}_{\tilde{\mathrm{g}}} \to \tilde{S}_{\tilde{\mathrm{g}}}$ being a lift of f to a finite regular cover $p : \tilde{S}_{\tilde{\mathrm{g}}} \to S_{\mathrm{g}}$. On the other hand, the results of 70ties of 20th century showed that the dynamics of a pseudo-Anosov homeomorphism of surface, e.g. its entropy, can be represented as the dynamics of quotient of finite sub-shift (a Markov partition). In this talk we pose a question whether a combination of two mentioned approaches could lead to an effective, computer assisted, method of distinguish whether there is a pseudo-Anosov piece in the canonical form of f and which let us to derive λ_f.

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Chaotic itinerancy. A set-oriented approach

Wojciech Jaworek[1],
joint work with Paweł Pilarczyk[1]

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Chaotic itinerancy (CI), brought to attention, among others, by Ikeda et al. [1] and Tsuda et al. [2], is a trajectory through high-dimensional state space, characterized by periods of ordered motion near quasi-attractors, followed by chaotic transitions. Possible mappings, in which CI can occur include coupled map lattice (CML) [3]. We study such mappings using combinatorial methods presented in [4], for which we partition state space into finite grid $\mathcal{X}$ of n-cubes and define multi-valued mapping $\mathcal{F}\colon \mathcal{X} \to \mathcal{X}$ that for each cube $x \in \mathcal{X}$ assigns the set of cubes $A_x \subset \mathcal{X}$. Mapping $\mathcal{F}$ can also be seen as a directed graph, with cubes as vertices and individual mappings between them as weighted edges. This gives us the general view of global dynamics, e.g. one can search for invariant sets by computing strongly connected components in graph $\mathcal{F}$. In this talk, I will briefly describe the model and mathematical tools used to analyze it, as well as potential directions for the continuation of this research.

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Mapping spaces of persistence diagram into the Hilbert space with controlled distortion

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The contents of the abstract. Stability is one of the most important properties of persistent homology. Similar inputs yield similar persistence diagrams. In this context, the space of persistence diagrams is typically equipped with the bottleneck metric. In order to apply statistical tools or further data analytic techniques to collections of persistence diagrams, we thus need to use a map from the space of persistence diagrams into a Euclidean or Hilbert space. In the past decade dozens of such maps have been proposed, including persistence landscape and persistence images. These maps are typically stable (Lipschitz). However, none of them has explicit lower bounds on distortion and hence they provide no control on the loss of information. In this talk we will present Lipschitz maps from certain spaces of persistence diagrams into Hilbert and Euclidean spaces with explicit distortion functions. The maps are fairly geometric, consisting essentially of bottleneck distances to specific landmark diagrams, and are thus easily implementable. The idea for the construction comes from the quantification of certain classical constructions in dimension theory.

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