Mapping spaces of persistence diagram into the Hilbert space with controlled distortion

Žiga Virk[1],
joint work with Atish Mitra[2]

[1] *University of Ljubljana, Slovenia*
[2] *Montana Tech, MT, USA*

The contents of the abstract. Stability is one of the most important properties of persistent homology. Similar inputs yield similar persistence diagrams. In this context, the space of persistence diagrams is typically equipped with the bottleneck metric. In order to apply statistical tools or further data analytic techniques to collections of persistence diagrams, we thus need to use a map from the space of persistence diagrams into a Euclidean or Hilbert space. In the past decade dozens of such maps have been proposed, including persistence landscape and persistence images. These maps are typically stable (Lipschitz). However, none of them has explicit lower bounds on distortion and hence they provide no control on the loss of information. In this talk we will present Lipschitz maps from certain spaces of persistence diagrams into Hilbert and Euclidean spaces with explicit distortion functions. The maps are fairly geometric, consisting essentially of bottleneck distances to specific landmark diagrams, and are thus easily implementable. The idea for the construction comes from the quantification of certain classical constructions in dimension theory.

[1] A. Mitra and Ž. Virk. Geometric embeddings of spaces of persistence diagrams with explicit distortions. *arXiv:2401.05298v1*, `https://doi.org/10.48550/arXiv.2401.05298`