Itineraries of nonoverlapping generalized Lorenz-like maps

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January 14–16, 2025

Our result states that if the rotation number of a nonoverlapping generalized Lorenz-like map is rational then all points in $[0,1)$ have eventually periodic itineraries with the same specific ordering of the periodic part of the itinerary uniquely determined by the rotation number. To be more precise, recall that a *generalized Lorenz-like map* is a map f of an interval $I = [0,1)$ to itself, for which there exists a point $c \in (0,1)$ such that f is nondecreasing on $I_L = [0,c)$ and on $I_R = [c,1)$. If $f(0) \geq f(1^-)$ then the generalized Lorenz-like map f is called *nonoverlapping*. The *itinerary* of x under f is the sequence $(s_0 s_1 s_2 \dots)$ where

$$s_j = s_j(x) = \begin{cases} 0 & \text{if } f^j(x) \in I_L, \\ 1 & \text{if } f^j(x) \in I_R. \end{cases}$$

For a point $x \in [0,1)$ and a positive integer n we will denote by $R(x,n)$ the number of integers $i \in \{0,\dots,n-1\}$ such that $f^i(x) \in I_R$. If the limit $\rho(x) = \lim_{n\to\infty} \frac{R(x,n)}{n}$ exists, we will call it the *rotation number* of x. Note that if f is a nonoverlapping (i.e., $f(1^-) \leq f(0)$) generalized Lorenz-like map, then all points have the same rotation number and we will denote it by $\rho(f)$ (see [2]). We are ready to formulate our theorem.

Let f be a nonoverlapping generalized Lorenz-like. Assume that $\rho(f) = p/q$, where p and q are coprime nonnegative integers. Then for all $x \in [0,1)$ there is $m \in \mathbb{N}$ such that for all $k \geq 0$ we have

$$s_{m+k}(x) = 0 \ \text{ if and only if } \ kp \pmod q < q - p.$$

[1] Piotr Bartłomiejczyk, Piotr Nowak-Przygodzki. Itineraries of nonoverlapping generalized Lorenz-like maps. *preprint.*

[2] F. Rhodes, Ch. L. Thompson. Rotation numbers for monotone functions on the circle. *J. London Math. Soc.* **34** (1986), 360–368.