Fixed point indices of iterates of an orientation-reversing homeomorphism at a fixed point which is an isolated invariant set

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Let f be an orientation-reversing homeomorphism of \mathbb{R}^m $(m \ge 3)$ with an isolated fixed point at 0 for each iterate of f. In [2], the authors examined all possible sequences of indices $\{ind(f^n, 0)\}_{n=1}^{\infty}$ for m = 3, under the additional assumption that $\{0\}$ is an isolated invariant set. They proved that, in this case, the indices have highly constrained forms.

In a more recent work [1], we show that if the assumption requiring $\{0\}$ to be an isolated invariant set is removed, the indices are bounded only by the Dold relations. Moreover, using our construction, we can realize all the forms determined in [2] such that $\{0\}$ is an isolated invariant set.

Building upon the techniques outlined in [1], we delve into an open problem concerning the possible forms of the sequence $\{ind(f^n, 0)\}_{n=1}^{\infty}$ under the assumption that 0 is an isolated invariant set and m > 3. This discussion is intended to expand on the result established in [2].

- Grzegorz Graff and Patryk Topór. Fixed point indices of orientationreversing homeomorphisms. arXiv:2409.08753 (2024), https://doi.org/10. 48550/arXiv.2409.08753
- [2] L. Hernídez-Corbato, P. Le Calvez and F. R. Ruiz del Portal. About the homological discrete Conley index of isolated invariant acyclic continua. *Geometry & Topology* 17 (2013), 2977–3026, https://doi.org/10.2140/gt.2013.17.2977