Surface homeomorphisms of algebraically finite type

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In [3] Nielsen investigated properties of surface mapping classes of algebraically finite type, defined to be represented by homeomorphisms that are either periodic or reducible and periodic outside an invariant system of circles. In other words, they have no pseudo-Anosov pieces in Nielsen–Thurston decomposition. The name "algebraically finite type" was motivated by Nielsen's conjecture that such classes can be defined purely algebraically as the ones that induce a map on the first homology group whose spectrum consists only of roots of unity (the latter classes are called quasi-unipotent). These two definitions do not coincide because of Thurston's construction of pseudo-Anosov map inducing the identity transformation. However, it is still an open question which symplectic transformations can be obtained from mapping classes of algebraically finite type. We will discuss this problem, important also from the point of view of dynamics. Da Rocha [1] showed that the classes containing Morse–Smale diffeomorphism and classes of algebraically finite type are the same. Some constructions in terms of Lefschetz numbers we provided in [2].

[1] L. F. da Rocha, *Characterization of Morse—Smale isotopy classes on surfaces*, Ergod. Th. & Dynam. Sys. 5 (1985), 107–122.

[2] G. Graff, W. Marzantowicz, Ł. P. Michalak, A. Myszkowski, *Every finite set of natural numbers is realizable as algebraic periods of a Morse–Smale diffeomorphism*, preprint (2024), arXiv:2408.12372.

[3] J. Nielsen, *Surface transformation classes of algebraically finite type*, Danske Vid. Selsk. Mat.-Fys. Medd. 21 no. 2 (1944), 89 pp.