## Analysis of global dynamics using Conley-Morse graphs

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I will talk about about a method for creating a database of global dynamics with the help of Conley-Morse graphs. It lets us create a map on a rectangular grid for selected ranges of parameters that is helpful for identifying interesting, non-trivial dynamics and qualitative changes of dynamics.

The method utilises combinatorial approach to dynamics, Morse sets and the Conley index. Therefore, it was possible to create a software that will carry out the computations for discrete- and continuous-time systems in a rigorous manner. The software is available here, together with a guide how to compile the software.

The method was used in [4] for an analysis of a two-dimensional model of a neuron introduced by Chialvo in 1995. We plan to use the method to further the research done in [5] about the discrete two–gene Andrecut–Kauffman model. I will also use the method in my Master's thesis to analyse tritrophic food chain models.

Model A 
$$\begin{cases} x' = a(x - x_0) - \alpha_1 xy, \\ y' = -by + \alpha_1 xy - \alpha_2 yz, \\ z' = -c(z - z_0) + \alpha_2 yz, \end{cases}$$
(1)

where  $a, b, c, x_0$  and  $z_0$  are positive parameters, and

Model B 
$$\begin{cases} x' = rx(1 - px) - \frac{\alpha_1 x}{1 + k_1 y} y, \\ y' = -by + \frac{\alpha_1 x}{1 + k_1 y} y - \alpha_2 yz, \\ z' = -c(z - z_0) + \alpha_2 yz, \end{cases}$$
(2)

where  $r, p, b, c, x_0$  and  $z_0$  are positive parameters.

- [1] Conley-Morse Graphs Computation software https://www.pawelpilarczyk. com/cmgraphs/
- [2] Kokubu H., Spendlove K. A User's Guide to the Conley-Morse Database. https://kellyspendlove.github.io/cmdbSurveyII.pdf
- [3] Arai, Z. et al., A Database Schema for the Analysis of Global Dynamics of Multiparameter Systems, SIAM J. Appl. Dyn. Syst. Vol. 8, No. 3 (2009), 757–789. https://doi.org/10.1137/080734935
- [4] Pilarczyk P., Graff G., Signerska-Rynkowska J. Topological-numerical analysis of a two-dimensional discrete neuron model. CHAOS 33 (2023), 043110. https: //doi.org/10.1063/5.0129859
- [5] Rosman M., Palczewski M., Pilarczyk P., Bartłomiejczyk A. Bistability and chaos in the discrete two-gene Andrecut-Kauffman model. https://doi.org/ 10.48550/arXiv.2411.16699